Analytic Calculation of Neutrino Mass Eigenvalues

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Abstract

Implicaion of the neutrino oscillation search for the neutrino mass square difference and mixing are discussed. We have considered the effective majorana mass m_{ee}, related for $\beta\beta_{0\nu}$decay. We find limits for neutrino mass eigen value m_i in the different neutrino mass spectrum,which explain the different neutrino data.

1 Introduction

The phenomenon of neutrino oscillation, impressive advance have been made to understand the phenomenology of neutrino oscillation through solar neutrino,atmospheric neutrino,reactor neutrino and accelerator neutrino experiment. The experimental research on the nature of neutrinos through terrestrial as well as extra-terrestrial approaches has finally confirmed the neutrino oscillation in atmospheric [1,2,3,4] solar [5,6,7,8,9], reactor [10, 11,12] and accelerator [13,14] neutrino sources, establishing that neutrinos have mass.

Furthermore, it is generally agreed that oscillations among three neutrino species are sufficient to explain the atmospheric, solar, reactor and accelerator neutrino puzzle. The neutrino oscillation experiments provide us with neutrino mass square differences, mixing angles and a possible hierarchy in the neutrino mass spectrum. The main physical goal in future experiment are the determination of the unknown parameter θ_{13} . In particular, the observation of δ is quite interesting for the point of view that δ related to the origin of the matter in the universe. One of the most important parameter in neutrino physics is the magnitude of mixing angle θ_{13} and CP phase δ. The oscillation data also sugges that the neutrinos may belong to either a normal hierarchy $(m_1 < m_2 < m_3)$ or an inverted hierarchy $(m_3 < m_1 < m_2)$. The data do not exclude the possibility that the mass of the light neutrino could be much larger than $\sqrt{\triangle_{31}}$, which would imply the possible existence of a quasi-degenerate neutrino mass spectrum $(m_1 \approx m_2 \approx m_3)$. On the other hand, the actual mass of neutrinos cannot be extracted from these data, only the study of tritium single β decay and nuclear neutrino-less double beta decay together can provide sharpest limits on the mass and nature of neutrinos. Neutrino oscillations, which only depend on mass square difference, give no information about the absolute value of the neutrino mass squared eigenvalues . Hence, there are various possibilities of neutrino hierarchy spectrums consistent with solar and atmospheric neutrino oscillation data. The mass eigenstates with eigen values m_i can be determined if the absolute value of effective mass of neutrino is exactly known. The current neutrino-less double beta decay experiments only provide the upper limit on effective Majorana neutrino mass $< m_{ee} >$ so that absolute scale of neutrino mass is not determined yet. Therefore, in the present work we have attempted to present a picture of neutrino mass spectrum in the case of normal, inverted and almost degenerate hierarchy of neutrino masses by taking some specific choices of effective mass.

In this paper, we will discuss the masses of the vacuum eigenstates m_1,m_2 and $m_3$for different neutrino mass spectrum, namely normal mass hierarchy $(m_1 < m_2 < m_3)$,inverted mass hierarchy $(m_3 < m_1 < m_2)$ and almost degenerate spectrum $(m_1 \approx m_2 \approx m_3)$.The present work is organized as follows.In Sec.2, we outline the neutrino oscillation parameters. In Sec.3, we have given the theoretical formalism to calculate mass eigenvalues m_i for above mentioned hierarchies. In Sec.4, we present the numerical results and Sec.5 is devoted to the conclusions.

2 Mixing Angles and Neutrino Mass Squared Differences

The first evidence is the observation of zenith-angle dependence of atmospheric neutrino defect [15] dependent of the atmospheric neutrino $\nu_\mu \rightarrow \nu_\mu$ transition with the mass difference and the mixing as

$$\Delta_{31} = (1-2) \times 10^{-3} eV^2, sin^2 2\theta_{23} = 1.0. \tag{1}$$

The second evidence is the solar neutrino deficit [16], which is consistent with $\nu_\mu \rightarrow \nu_\tau/\nu_e$ transition. The SNO experiments [17] are consistent with the standard solar model [18] and strongly suggest the LMA solution.

$$\Delta_{21} = 7 \times 10^{-5} eV^2, sin^2 2\theta_{12} = 0.8. \tag{2}$$

Solar neutrino experiments (Super-K, GALLEX, SAGE, SNO and GNO) show the neutrino oscillations, neutrino oscillation provide the most elegant explanation of all the data [19].

$$\Delta_{solar} = 7^{+5}_{-1.3} \times 10^{-5} eV^2, \tag{3}$$

$$tan^2\theta_{solar} = 0.4^{+0.14}_{-0.1}. \tag{4}$$

Atmospheric neutrino experiments (Kamiokande, Super-K) also show the neutrino oscillation. The most excellent fit to the all data [19].

$$\Delta_{atmo} = 2.0^{+1.0}_{-0.92} \times 10^{-3} eV^2, \tag{5}$$

$$sin^2 2\theta_{atmo} = 0.4^{+0.14}_{-0.10}. \tag{6}$$

The CHOOZ reactor experiment [20] gives the upper bound of the third mixing angle θ_{13}as

$$sin^2\theta_{13} < 0.20 \qquad for \qquad |\Delta_{31}| = 2.0 \times 10^{-3} eV^2, \tag{7}$$

$$sin^2\theta_{13} < 0.16 \qquad for \qquad |\Delta_{31}| = 2.5 \times 10^{-3} eV^2,, \tag{8}$$

$$sin^2\theta_{13} < 0.14 \qquad for \qquad |\Delta_{31}| = 3.0 \times 10^{-3} eV^2, \tag{9}$$

at the 90 % CL. The CP phase δ has not been constrained. The future neutrino experiments plan to measure the oscillation parameters precisely.

3 Effective Majorona Mass of Electron Neutrino

In the presence of three flavour neutrino mixing the electron neutrino is combination of mass eigenstate, ν_i with eigenvalue m_i

$$\nu_e = \sum U_{ej}\nu_{ij} \;\; i = 1, 2, 3. \tag{10}$$

Here $U_{ei}$are the elements of the mixing matrix,which relates the flavour states the the mass eigenstates. The $\beta\beta_{0\nu}$ decay rate is determined by the effective Majorana mass of the electron neutrino m_{ee}. Under the assumption of three flavour neutrino mixing of neutrino, the effective Majorana neutrino mass m_{ee} is

$$|m_{ee}| = |\sum |U_{ej}|^2 e^{i\phi_j} m_j|$$

$$= |c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 e^{i\phi_2} m_2 + s_{13}^2 e^{i\phi_3} m_3|. \tag{11}$$

Where

$|U_{ej}|$,j=1,2,3 are the absolute values of the elements of the first row of neutrino mixing matrix,

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23-}s_{12}s_{13}s_{23}e^{i\delta} & c_{23}s_{13} \end{pmatrix},$$

here $c_{ij} = cos\theta_{ij}$,$s_{ij} = sin\theta_{ij}$. The angle ϕ_2,ϕ_3 are the two majorana CP phase. The masses of the vacuum eigenstates are takes to be m_1,m_2 and m_3. On squaring eq(11),

$$|m_{ee}|^2 = A^2m_1^2+\mathbf{B}^2m_2^2+C^2m_3^2+2ABm_1m_2cos\phi_2+2ACm_1m_3cos\phi_3+2BCm_3m_2cos(\phi_2-\phi_3), \tag{12}$$

where $A = c_3^2c_2^2$,$B = c_3^2s_2^2$, $C = s_3^2$

4 Numerical Results

In table-1,table-3 and table-3, we list neutrino mass eigenvalue m_i in case of different neutrino mass spectrum, using the best fit value given in ref[21].We have varied m_{ee} from 1- 10 meV and 10-100 meV in case of normal and inverted hierarchy respectively. For degenerate case,$m_1 \approx m_2 \approx m_3$, and considering the analysis of the Sloan Digital Survey Data and WMAP data [22], Wilkinson Microwave Anisotropy Probe (WMAP) and 2 degree Field Galaxy Redshift Survey (2dFGRS) data [23], we have varied m_{ee} from 100-600 meV. For normal hierarchy, m_1 varies from 0.0002eV - 0.01eV and m_2 varies from 0.009 eV- 0.013eV, while variation in m_3 is from 0.049-0.056 eV. The variation in m_3 is quite less in comparison to m_2 and m_1. In case of inverted hierarchy, m_1 and m_2 both vary from 0.01-0.263 eV while m_3 varies from 0.047-0.259 eV for considered values of m_{ee}. For 10-30 meV range of m_{ee}, m_3 values are higher than m_1 and m_2, while for 40-100 meV range of m_{ee}, m_3 value is lower than m_1 and m_2. The almost same variation for m_1 and m_2 also confirms the consideration of m_1, m_2, for inverted hierarchy. In case of AD hierarchy, all the m_1, m_2, m_3 values lie in the range from 0.26-1.74 eV. We compute the neutrino mass eigenvalue using eq (11). We have taken normal mass hierarchy $\Delta_{31} > 0$, inverted mass hierarchy$\Delta_{31} < 0$ and almost degenerated case. For simplicity, we have set the majorana phases $\phi = 0^o, 180^o$.

Mass Hierarchy	m_ν	Mass Eigenstate	Majorana Phases	Majorana Phases
	(eV)	(eV)	$\phi_2 = 0^0, \phi_3 = 180^0$	$\phi_3 = 0^0, \phi_2 = 180^0$
Normal	0.010	m_1	1.0×10^{-2}	2.62×10^{-2}
		m_2	1.33×10^{-2}	2.77×10^{-2}
		m_3	5.05×10^{-2}	5.60×10^{-2}
	0.009	m_1	8.89×10^{-3}	2.36×10^{-2}
		m_2	1.25×10^{-2}	2.52×10^{-2}
		m_3	5.03×10^{-2}	5.49×10^{-2}
	0.008	m_1	7.76×10^{-3}	2.11×10^{-2}
		m_2	1.17×10^{-2}	2.28×10^{-2}
		m_3	5.01×10^{-2}	5.38×10^{-2}
	0.007	m_1	6.61×10^{-3}	1.86×10^{-2}
		m_2	1.09×10^{-2}	2.05×10^{-2}
		m_3	4.99×10^{-2}	5.29×10^{-2}
	0.006	m_1	5.43×10^{-3}	1.61×10^{-2}
		m_2	1.03×10^{-2}	1.83×10^{-2}
		m_3	4.98×10^{-2}	5.20×10^{-2}
	0.005	m_1	4.21×10^{-3}	1.36×10^{-2}
		m_2	9.70×10^{-3}	1.62×10^{-2}
		m_3	4.97×10^{-2}	5.13×10^{-2}
	0.004	m_1	4.94×10^{-3}	1.13×10^{-2}
		m_2	9.22×10^{-3}	1.42×10^{-2}
		m_3	4.96×10^{-2}	5.08×10^{-2}
	0.003	m_1	1.61×10^{-3}	8.98×10^{-2}
		m_2	8.89×10^{-3}	1.25×10^{-2}
		m_3	4.95×10^{-2}	5.03×10^{-2}
	0.002	m_1	1.87×10^{-4}	6.84×10^{-3}
		m_2	8.74×10^{-3}	1.11×10^{-2}
		m_3	4.95×10^{-2}	4.99×10^{-2}
	0.001	m_1	1.35×10^{-3}	4.85×10^{-2}
		m_2	8.84×10^{-3}	9.99×10^{-2}
		m_3	4.95×10^{-2}	4.97×10^{-2}

Table 1: Neutrino mass eigenvalue for normal hierarchy mass spectrum. Input value are given in ref[21]

Mass Hierarchy	m_ν	Mass Eigenstate	Majorana Phases	Majorana Phases
	(eV)	(eV)	$\phi_2 = 0^0, \phi_3 = 180^0$	$\phi_3 = 0^0, \phi_2 = 180^0$
Inverted	0.10	m_1	1.04×10^{-1}	2.63×10^{-2}
		m_2	1.04×10^{-1}	2.63×10^{-2}
		m_3	9.18×10^{-2}	2.59×10^{-2}
	0.09	m_1	9.33×10^{-2}	2.37×10^{-2}
		m_2	9.38×10^{-2}	2.37×10^{-2}
		m_3	7.98×10^{-2}	2.32×10^{-2}
	0.08	m_1	8.29×10^{-2}	2.11×10^{-2}
		m_2	8.33×10^{-2}	2.11×10^{-2}
		m_3	6.73×10^{-2}	2.05×10^{-2}
	0.07	m_1	7.24×10^{-2}	1.84×10^{-2}
		m_2	7.29×10^{-2}	1.85×10^{-2}
		m_3	5.38×10^{-2}	1.78×10^{-2}
	0.06	m_1	6.18×10^{-2}	1.58×10^{-2}
		m_2	6.24×10^{-2}	1.58×10^{-2}
		m_3	3.85×10^{-2}	1.51×10^{-2}
	0.05	m_1	5.11×10^{-2}	1.32×10^{-2}
		m_2	5.19×10^{-2}	1.32×10^{-2}
		m_3	1.65×10^{-2}	1.23×10^{-2}
	0.04	m_1	4.11×10^{-2}	1.06×10^{-1}
		m_2	4.19×10^{-2}	1.06×10^{-1}
		m_3	2.56×10^{-2}	9.43×10^{-2}
	0.03	m_1	3.09×10^{-2}	8.01×10^{-2}
		m_2	3.22×10^{-2}	8.10×10^{-2}
		m_3	3.71×10^{-2}	6.38×10^{-2}
	0.02	m_1	2.07×10^{-2}	5.46×10^{-2}
		m_2	2.25×10^{-2}	5.51×10^{-2}
		m_3	4.38×10^{-2}	2.54×10^{-2}
	0.01	m_1	1.01×10^{-2}	2.67×10^{-2}
		m_2	1.34×10^{-2}	2.81×10^{-2}
		m_3	4.73×10^{-2}	4.03×10^{-2}

Table 2: Neutrino mass eigenvalue for inverted hierarchy mass spectrum. Input value are given in ref[21]

Mass Hierarchy	m_ν	Mass Eigenstate	Majorana Phases	Majorana Phases
	(eV)	(eV)	$\phi_2 = 0^0, \phi_3 = 180^0$	$\phi_3 = 0^0, \phi_2 = 180^0$
Almost degenerate	0.6	m_1	1.58	1.74
		m_2	1.58	1.74
		m_3	1.58	1.74
	0.5	m_1	1.32	1.45
		m_2	1.32	1.45
		m_3	1.32	1.45
	0.4	m_1	1.06	1.16
		m_2	1.06	1.16
		m_3	1.06	1.16
	0.3	m_1	0.79	0.87
		m_2	0.79	0.87
		m_3	0.79	0.87
	0.2	m_1	0.53	0.58
		m_2	0.53	0.58
		m_3	0.53	0.58
	0.1	m_3	0.26	0.29
		m_2	0.26	0.29
		m_3	0.26	0.29

Table 3: Neutrino mass eigenvalue for degenerated mass spectrum. Input value are given in ref[21]

5 Conclusions

Future and present search for neutrinoless double beta decay purpose at probing lepton number violation and the Majorona nature of neutrinos with remarkable precession.Several experimental programs is currently under discussion. We find the mass eigenvalue $m_i$in case of normal hierarchy,inverted mass hierarchy and almost degenerate neutrino mass spectrum. By taking $m_{ee} = (0.010 - 0.001)$ eV as reference value, for normal mass hierarchy spectrum.We have predicted that m_1 varies from 0.0002eV - 0.01eV and m_2 varies from 0.009 eV- 0.013eV, while variation in m_3 is from 0.049-0.056 eV. In case of inverted hierarchy, m_1 and m_2 both vary from 0.01-0.263 eV while m_3 varies from 0.047-0.259 eV for considered values of m_{ee}. For 10-30 meV range of m_{ee}, m_3 values are higher than m_1 and m_2, while for 40-100 meV

range of m_{ee}, m_3 value is lower than m_1 and m_2. The almost same variation for m_1 and m_2 also confirms the consideration of m_1, m_2, for inverted hierarchy. In case of AD hierarchy, all the m_1, m_2, m_3 values lie in the range from 0.26-1.74 eV. We have calculated the mass eigenvalue m_1, m_2 and m_3 for all the three mass spectrum considered by taking some specific choice of effective neutrino mass depending on the type of mass spectrum. The ordering of mass states depends on choice of m_{ee}, hence precise determination of m_{ee} from single beta decay experiment (Tritium beta decay) and future neutrino-less double beta decay experiments will make the picture of mass spectrum clear.

References

[1] Super Kamiokande Collaboration, R. Wendell *et al.*,Phys. Rev. **D** 81, 092004 (2010).

[2] Super-Kamiokande Collaboration,Y. Ashie *et al.*, Phys. Rev. **D** 71, 112005 (2005).

[3] Super-Kamiokande Collaboration, Y. Fukuda *et al.*,Phys. Rev. Lett. 82, 1810 (1999); Phys. Rev. Lett. 82, 2430 (1999); Phys. Rev. Lett. 82, 2644 (1999); Phys. Rev. lett. 81, 1562 (1998).

[4] Macro Collaboration, A. Surdo *et al.*, Nucl. Phys. Proc. Suppl. 110, 342 (2002).

[5] Soudan Collaboration,M. Sanchez *et al.*, Phys. Rev. D 68, 113004 (2003).

[6] Super Kamiokande Collaboration, J.P. Cravens *et al.*, Phys. Rev.D 78, 032002 (2008).

[7] K. Abe *et al.*, arXiv: hep-ex1010.0118.

[8] B. T. Cleveland *et al.*, Astrophys. J 496, 505 (1998); C. Cattadori, N. Ferrari and L. Pandola, Nucl. Phys. B (Proc. Suppl.) 143, 3 (2005).

[9] Super-Kamiokande Collaboration, Y. Fukuda *et al.*,Phys. Lett. B539, 179 (2002); Phys. Rev. D 73, 112001 (2006).

[10] SNO Collaboration,Q. R. Ahmad *et al.*, Phys. Rev. Lett. 89, 011301 (2002).

[11] A. Gando *et al.*,arXiv: hep-px/1009.4771.

[12] KamLand Collaboration,K. Eguchi *et al.*, Phys. Rev. Lett. 90, 021802 (2003), hep-ex/0212021.

[13] KamLand Collaboration,T. Araki *et al.*, Phys. Rev. Lett. 94, 081801 (2005).

[14] K2K Collaboration, E. Aliu *et al.*, Phy. Rev. Lett. 94, 081802 (2005).

[15] Super-Kamiokande Collaboration, Y. Ashie *et al.*,Phys. Rev **D** **71**;112005 (2005).

[16] M. Freund, P. Hunder, M. Lindner, Nucl. Phys **B 615** ;331-357 (2001).

[17] A. Cervera *et al.*,Nucl. Phy. **B** 579, 17 (2000).

[18] Super-Kamiokande Collaboration, Y. Fukuda *et al.*,Phys. Rev. Lett **81** 1562 (1998).

[19] W. Hampel, Phys.Lett **B 4**47,127 (1999).

[20] CHOOZ Collaboration, C. Apollonio *et al.*, Eur. Phy. **J** (**27,33**) (2003).

[21] T. Schwetz, M.A. Tortola and J.W.F. Valle; New J. Phys., 10, 113011 (2008).

[22] [1] C.L. Bennet et al., astro-ph/0302207; D.N. Spergel et al., astro-ph/0302209.

[23] M. Tegmark et al., astro-ph/0310723.